XV. gimnazija

IB MYP

Physics Project

Dependence of Pressure on Temperature

Criteria B, C

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Introduction

The task given was to investigate the dependance of pressure on temperature in a gas. The dependency should be found on the temperature in degrees Celsius and presented as a formula. According to the ideal gas law, which states that the product of pressure (p) and volume (V) equals the product of the gas quantity /number of moles(n), the ideal gas constant (R) and temperature in Kelvins (T) if volume and gas quantity are constant (ideal gas constant is always constant):

$$pV = nRT$$

The equation may be rewritten to represent the constant ($const.$):

$$\frac{p}{T} = const.$$

Such a relationship would mean that an increase in temperature would result in a linear increase of pressure. It is linear rather than proportional as there would be an offset due to Kelvin-Celsius difference, as the ideal gas law uses Kelvins, which equal the value in Celsius plus 273.15. Furthermore, this may be translated to the physical point of view, where increasing temperature is a measure of internal energy which is partially formed of kinetic energy. Thus, molecules would move around more at higher temperatures and hit the sides of the container with higher force and in greater number, causing an increase of pressure, as pressure equals the quotient of force and area (which is constant in a container). Thus, the hypothesis is that the dependence of pressure on temperature would be linear with an offset of 273.15°C.

Materials and Method

The method involves changing the temperature of a gas (air) and observing how the pressure changes if volume and gas quantity are constant. As the volume is constant, the thermodynamic process involved is isochoric. In order to see if there is a dependance and if there is what kind, an absolute pressure-temperature graph is drawn (since temperature is the independent variable). Then, if the dependency appears linear or proportional, the equation of dependence may be simply obtained by taking two points and calculating the gradient accordingly. The offset is then found by looking at the pressure axis intercept. However, if the dependance appears inverse-proportional, then another graph of pressure-temperature^{-1} is drawn and the dependance is derived for that graph in the same method as if it was a linear/proportional.

For measuring the difference between external air pressure and the air pressure a manometer is used with a precision of 0.5kPa. As the manometer does not provide absolute air pressure measurement which are needed, meaning that the local air pressure available online[1] needs to be added upon each manometer measurement. The manometer itself is installed on cap pressed on an Erlenmeyer flask which is filled with air at atmospheric pressure. To heat the gas, a water boiler and approximately 1L of water is needed. Furthermore, a container to pour the water which is sufficiently taller and wider to fit the Erlenmeyer flask completely in is needed, as well as a digital thermometer with a precision of 0.1°C and a stand two clamps.

Before beginning the method the air pressure should be checked using the selected website and noted down. Furthermore, the manometer is calibrated. Firstly, the Erlenmeyer flask with the manometer is placed inside the container and fixed with a clamp such that it is equally distanced from the bottom as it is from the sides at the flask's widest point (Figure 1). This becomes relevant when the water is poured in, as heat should be equally distributed throughout the water and, subsequently, the flask to allow for measurements of the water's temperature near the flask to be considered as the temperature of the gas as a whole within the flask. Then, the thermometer is attached onto the other clamp on the stand and positioned such that the thermocouple is as close to the flask as possible and as deep inside the container as possible, as the temperature of water closest to the flask is closest to that of the air inside due to closer proximity to the point of heat conduction (Figure 2). Then, 1L of water is poured into the water boiler and the heater is turned on until water reaches the boiling point and turns off automatically. Then, hot water is poured into the container surrounding the Erlenmeyer flask such that the flask is completely submerged under the water, with the manometer only protruding above the water, so that the heat received by the gas from the air is as even within the gas itself as possible, so that the temperature measurement of the water represents the air temperature as truthfully as possible (Figure 3). Initially, the gas will be heated by the water. Due to the sudden heating, rapid changes of the pressure readings are to be expected, so sufficient time should be used for the readings to stabilize. Thus, to leave a significant amount of time for the temperatures to equalize and to have enough measurements, the first temperature to start measurements should be 85.0°C. For each 5.0°C drop in temperature, the respective pressure value is recorded. After thermal equilibrium is reached with the environment and the water temperature ceases to decrease, the water is poured into the sink and the apparatus is disassembled.

The independent variable is temperature as it is being changed by the cooling of previously heated water. Accordingly, the pressure is being measured at set intervals. As the gas (air) which is used in the relationship of temperature and pressure is being investigated is in a sealed container, the volume and gas quantity are being controlled (not with measurement but rather with the physical

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[1] "DHMZ - Croatian Meteorological and Hydrological Service." *Meteo.hr*, 2021, meteo.hr/naslovnica_aktpod_en.php?tab=aktpod.

setup). It is important to control these quantities as they also affect the pressure of the gas according to the ideal gas formula. Furthermore, changing volume would mean that the frequency of particles hitting the side and applying pressure would decrease. The same applies for gas quantity. Atmospheric temperature measurements do not need to be taken as the temperature as close as possible to the gas is already being taken.

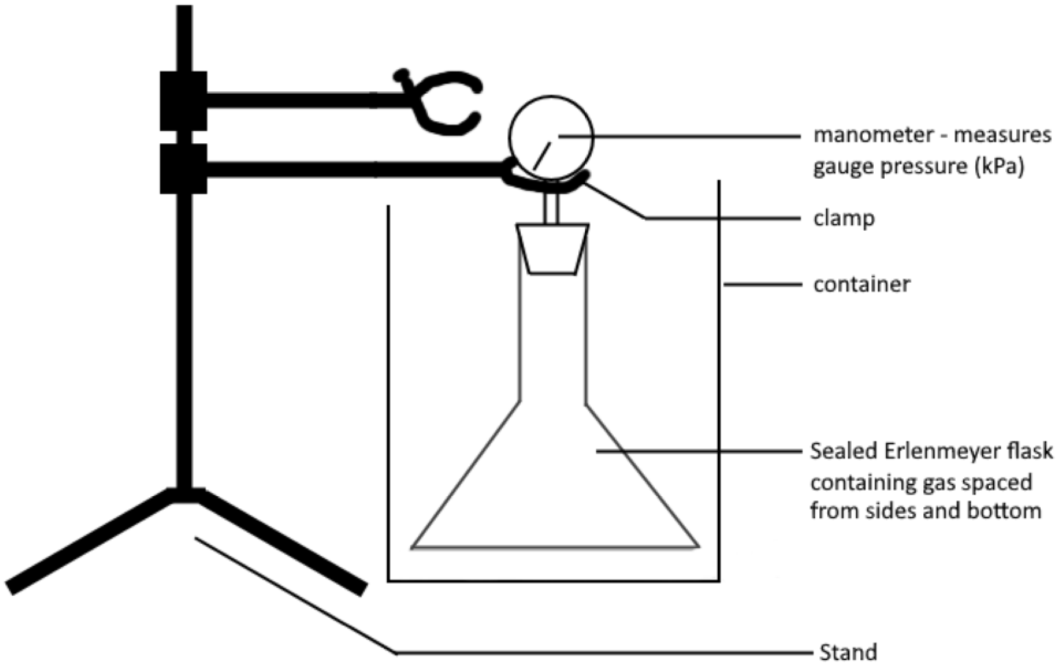

Figure 1 The Sealed Erlenmeyer flask is placed inside the container

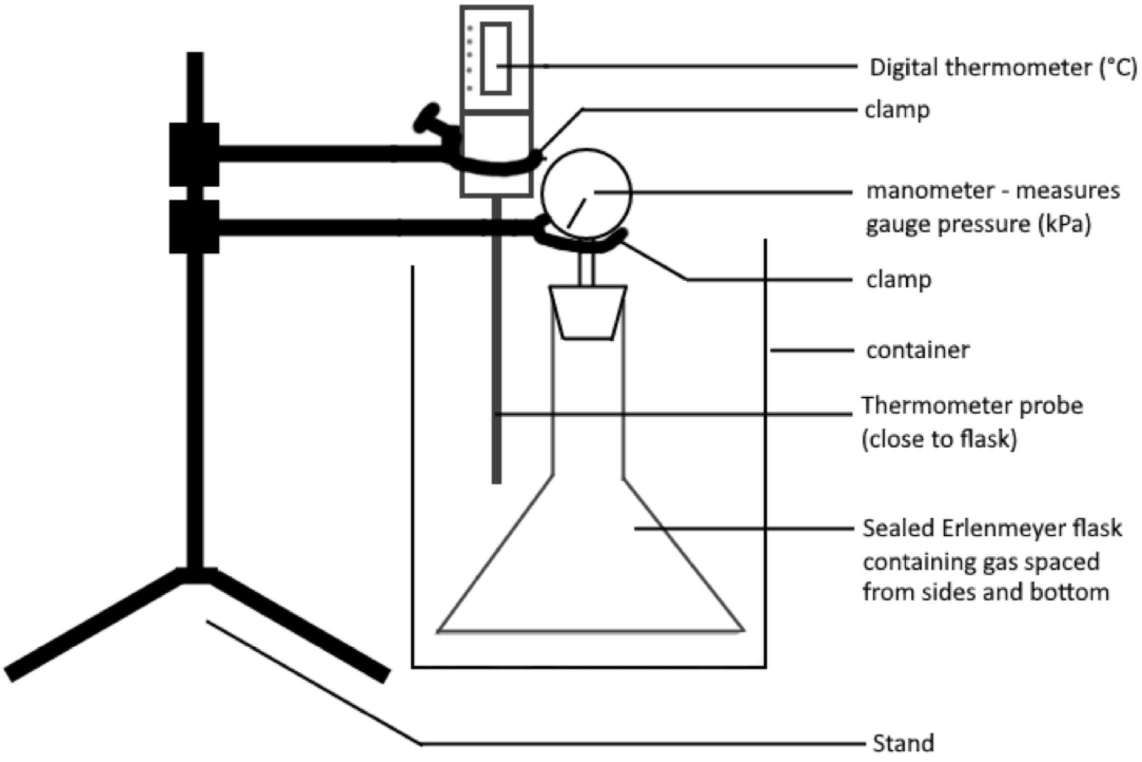

Figure 2 Thermometer is attached to the stand and installed according to the method

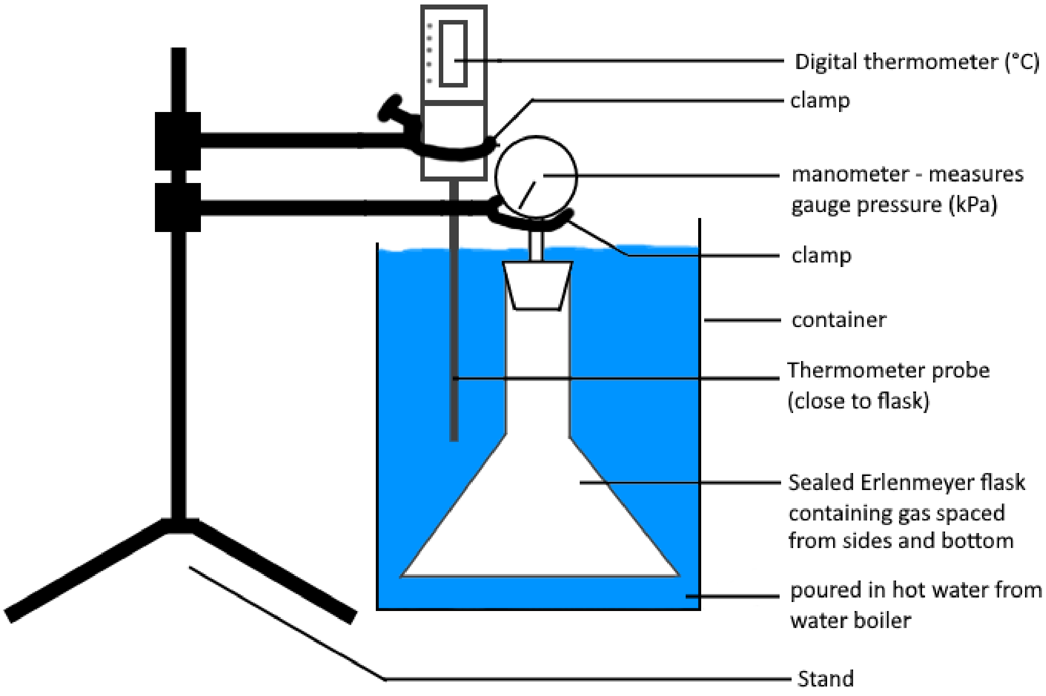

Figure 3 Boiling water from the water heater is is pured in to fully cover the flask containing the gas

After the execution of the method, temperature (t) and gauge pressure (p_g) values are obtained. Absolute pressure (p_a) values are obtained by adding the gauge pressure to online-found atmospheric air pressure (p_0). However, if absolute pressure values were used the drawn graph would have a very high vertical offset which would severely constrict the precision of drawing and reading values. This is addressed further on. A gauge pressure-temperature graph is drawn to observe the distribution of points. If the distribution appears to have an inverse proportional dependance, a second graph is drawn, namely a pressure by inverse temperature ($1/t$) graph. In either case, after all the point are drawn, a best fit line is drawn. From the lines two points are taken (P_1 and P_2):

$$P_1\left(t_1, p_{g_1}\right)$$

$$P_2\left(t_2, p_{g_2}\right)$$

The slope (a) which is needed to obtain the equation of dependance is found by dividing the change in absolute pressure with temperature or 1/temperature.

$$a = \frac{\Delta p_g}{\Delta t}$$

$$a = \frac{p_{g_2} - p_{g_1}}{t_2 - t_1}$$

or, in the case of a parabolic curve on the first graph:

$$a = \frac{\Delta p_g}{\Delta t^{-1}}$$

$$a = \frac{p_{g_2} - p_{g_1}}{{t_2}^{-1} - {t_1}^{-1}}$$

In either case, the pressure axis intercept (b) is determined by reading it from the pressure axis. The atmospheric pressure is added upon this offset (the one which was not included int the pressure) to represent the absolute pressure which is he relevant variable. The values are then substituted into the general equation of a straight line:

$$p_a = a \cdot t + b + p_0$$

or, in the case of a parabolic curve on the first graph:

$$p_a = a \cdot t^{-1} + b + p_0$$

Results

Through the execution of the method, the following results were obtained (Table 1):

n	t/°C	p_g/kPa
1	32.0	1.5
2	41.0	4.5
3	50.0	7.5
4	53.0	8.5
5	59.0	10.5
6	62.0	11.5
7	73.0	15.5
8	83.0	18.5
9	92.0	21.5
10	95.0	22.5

Table 1 Results Table

As described in the method, all the points are graphed on a gauge pressure by temperature graph. The graph is seen as Figure 4, attached at the end of the report. After the points are drawn, the best fit line is also drawn. Observing the line indicates a linear relationship, and the appropriate method for this case will be applied. Two points spaced significantly apart (so that a larger sample is represented) are taken from the best fit line (P_1 and P_2):

$$P_1(40.0\text{°C}, 4.0kPa\ )$$
$$P_2(84.0\text{°C}, 19.0\text{kPa})$$

The gradient is found by dividing the change in gauge pressure with the change in temperature.

$$a = \frac{p_{g_2} - p_{g_1}}{t_2 - t_1}$$

$$a = \frac{(19.0 - 4.0)kPa}{(84.0 - 40.0)°C}$$

$$a = \frac{(19.0 - 4.0)kPa}{(84.0 - 40.0)°C}$$

$$a = \frac{15.0kPa}{44.0°C} = 0.341\frac{kPa}{°C}$$

In division, the significant digits of the result are those of the number which has the lowest number of significant digits. In this case, that is three significant digits. A difficulty is encountered on the graph when attempting to find the gauge pressure axis intercept, as the intersection would happen somewhere outside the bounds of the paper. As such, the intersection must be derived mathematically. The equation of line for gauge pressure with the gradient is firstly made:

$$p_g = \left(0.341\frac{kPa}{°C}\right) \cdot t + b$$

The equation is rewritten to represent the pressure axis intercept. One of the previously used points (P_1) values are then substituted into the equation:

$$b = p_g - \left(0.341\frac{kPa}{°C}\right) \cdot t$$

$$b = 4.0kPa - \left(0.341\frac{kPa}{°C}\right) \cdot 40.0\text{°C}$$

$$b = 4.0kPa - \left(0.341\frac{kPa}{°C}\right) \cdot 40.0\text{°C}$$

$$b = 4.0kPa - 13.6kPa$$

$$b = -9.6kPa$$

Accordingly, the gauge pressure axis intercept is substituted into the gauge pressure equation of dependance:

$$p_g = \left(0.341\frac{kPa}{°C}\right) \cdot t - 9.6kPa$$

The equation of dependance for absolute pressure is obtained by adding the atmospheric pressure to the pressure axis intercept. At the day the data was given (14.5.2021), the regional air pressure was 1008.9hPa. When converted to kilopascals, that would be 100.89kPa. The value is rounded to the same precision as the gauge pressure intercept.

$$p_0 = 1008.9\text{hPa} = 100.89\text{kPa} = 101.0\text{kPa}$$

The atmospheric pressure is then added to the gauge pressure axis intercept to obtain the dependence for absolute pressure:

$$p_a = \left(0.341\frac{kPa}{°C}\right) \cdot t - 9.6kPa + 101.0\text{kPa}$$

$$p_a = \left(0.341\frac{kPa}{°C}\right) \cdot t + 91.4kPa$$

The dependance of pressure on temperature is as such linear, as the hypothesis postulated. In order to answer to the question concerning the temperature axis intercept, the equation of dependence is firstly rewritten to represent temperature:

$$t = \frac{p_a - 91.4kPa}{0.341\frac{kPa}{°C}}$$

Zero is substituted as the pressure to find the temperature axis intercept (t_0):

$$t_0 = \frac{0 - 91.4kPa}{0.341\frac{kPa}{°C}}$$

$$t_0 = -268.0°C$$

Discussion

Through the execution of the method and appropriate analysis of the results, the following equation of dependence of pressure on temperature on degrees Celsius was obtained:

$$p_a = \left(0.341\frac{kPa}{°C}\right) \cdot t + 91.4kPa$$

The dependance according to the obtained equation is linear, as the pressure at 0°C is not 0kPa as the pressure axis intercept is not zero. As such, the hypothesis which was based on the ideal gas law is proven correct. The obtained equation is essentially a variation of Gay-Lussac's law (one of the constituent formulas of the ideal gas law), which states that the pressure (p) is directly proportional to the gas temperature in Kelvins (T), if the volume and gas quantity are kept constant. As such it may be denoted that the quotient of pressure with temperature in Kelvins is a constant (k):

$$\frac{p}{T} = k$$

However, the equation of dependance obtained above does not equal to the law above. A significant difference between the obtained data and the law from literature is the unit of temperature – obtained data is in degrees Celsius, while the law assumes temperature in Kelvins. The offset between absolute zero (the start of the Kelvin scale) with zero degrees Celsius is 273.15K. The differences between one degree Celsius and Kelvin is the same, but the reference zero varies between the freezing point of water and the absolute zero. When zero kilopascals is inputted as the pressure, the temperature of -268.0°C is obtained (see Results). This value signifies the point at which temperature the pressure of the gas would be nonexistent. As absolute pressure cannot go into the negative, this represents sort of the starting point of the line. The value of 268.0°C is very similar to the theoretical difference and very likely originates from the difference in units due to similarity to theoretical values. If this was accounted for when plotting the graph and using Kelvins instead, all temperature measurements in degrees Celsius would be shifted by 273.2°C. As such, the temperature axis intercept in Kelvins (T_0) could be found by adding 273.2°C to the current value:

$$T_0 = t_o + 273.2\text{°C}$$

$$T_0 = -268.0\text{°C} + 273.2\text{°C} = 5.2\text{K}$$

The obtained temperature axis intercept is very close to zero degrees Kelvin when compared to the pervious difference. The obtained graph in this case would still have the same slope (as the differences between points still stay the same), but the pressure axis intercept would be different, not to mention the already established new temperature axis intercept of 5.2K. In order to find the offset (c) in this situation, the equation of a straight line with the same slope as before is firstly rewritten to represent the pressure axis intercept:

$$p_a = \left(0.341 \frac{kPa}{K}\right) \cdot T + c$$

$$c = p_a - \left(0.341 \frac{kPa}{K}\right) \cdot T$$

One of the points previously used points' values are then substituted. The point's temperature in degrees Celsius is converted to degrees Kelvin by adding 273.2°C. Furthermore, as the points initially represented only gauge pressure to which the atmospheric pressure was added later on, the atmospheric pressure of 101.0kPa is added to the gauge pressure value. This point previously in Celsius (P_1) is represented now as ${P_1}_K$:

$${P_1}_K(40.0°\text{C} + 273.2°\text{C}, 4.0kPa + 101.0kPa\,)$$

$${P_1}_K(313.2K, 105.0kPa\,)$$

The values are then substituted into the equation for the pressure axis intercept:

$$c = 105.0kPa - \left(0.341 \frac{kPa}{K}\right) \cdot 313.2K$$

$$c = -1.8kPa$$

The intercept for the temperature axis in Kelvins is substituted into the equation of a straight line for the graph in Kelvins:

$$p_a = \left(0.341 \frac{kPa}{K}\right) \cdot T - 1.8kPa$$

The pressure axis intercept value of -1.8kPa in the graph in degrees Kelvin is much smaller to the previously significantly larger 91.4kPa. The previously discussed temperature intercept in degrees Kelvins is also much closer to the origin than the temperature intercept in degrees Celsius. Subsequently, the graph in Kelvins would look much more similar to a proportional relationship than its counterpart in degrees Celsius. As such, this is much closer to theoretical values from Gay-Lussac's law, but there is still an offset, likely resulting from imprecision in measuring and deficiencies in the method.

Firstly, there might not be enough time initially to allow the temperature of the surrounding fluid to equalize with that of the air within the sealed flask. As such, the temperature used for calculations might not be the actual temperature of the gas. As such, it would affect the final dependency. The only way to be sure of temperature equalization would be to measure the temperature from within the sealed gas contained by placing a probe from a digital thermometer within. Then, the obtained value could be compared to the water temperature. Water temperature would still need to be taken as it would indicate uniform temperature distribution from all sides of the flask, and subsequently, the temperature of the gas. As such, adding a thermometer within the flask may be an improvement to the method. Furthermore, due to the insulative property of glass, the temperature within the glass flask may not be equal to that of the water outside – the change of temperature in the water would be delayed in the temperature of the gas. To ensure the temperatures be as equal as possible between the gas and water, a more heat conductive material such as copper should be used for the flask to improve on the method. There is an issue within the water container itself. Transfer of heat from the container to the air would happen in the greatest quantity near the top of the flask, as there is no insulative material from the container that would diminish the transfer of heat. The top water would cool the first, causing it to become colder and subsequently to become less dense and begin to sink (convection). Warmer water would come to the top and the process would repeat. Such uneven movement of different-temperature water would cause uneven temperature of water and within the gas. A way to solve this issue would be to have a mixer inside the flask which would evenly stir the water and cause any differences in temperature to be evened out inside the container.

The heat from the water in the current setup would conduct into the manometer itself. This may impact the measurements by causing thermal expansion of the various elements within the manometer and affecting its behavior. Furthermore, the manometer itself includes a certain amount of air, and as the manometer is not submerged, this air would not be at equal temperature as the rest of the sample. Cooling would occur within the manometer and cause similar convection to that which may happen in the water. These effects are however likely negligible in the current setup due to the relatively small temperatures at which thermal expansion is not so great. However, care should be taken to select manometers which have a high operable heat range and as low as possible internal volume in which air would have a chance to cool.

Another limitation of the method would be the fact that water is used as the medium which would heat the air. This is limiting in the melting and boiling points of water which are relatively small when compared to some other substances. A small temperature range limits what measurements may be taken and may lead to a wrong conclusion on the dependance type (this is addressed further on).

Water does have advantages such as high thermal capacity and conductivity which would contribute to even temperature distribution. Using, for example, Gallium[2] as a medium which has a comparatively very large temperature range between melting and boiling point (30°C to 2400°C), and is also a metal, meaning it has a much greater heat conductivity than water (40.6 W/(m·K) compared to 0.6 W/(m·K)) would yield a broader temperature range and contribute even more to even temperature distribution. In the case this is applied, many materials would need to be changed to allow for the increased temperature and likely a manometer with a greater scale would be needed. Proper heating methods and control would need to be made. This would be more akin to an experiment redesign than a mere improvement.

Lastly, the atmospheric pressure found online may have not represented the actual pressure at the place of the experiment due to local variations in weather. A simple improvement of using a barometer at the place of the experiment to measure the atmospheric pressure may be made. Some of these mentioned improvements which are most feasible to apply without completely changing the apparatus are shown in Figure 5.

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[2] Wikipedia Contributors. "Gallium." *Wikipedia*, Wikimedia Foundation, 27 Apr. 2021, en.wikipedia.org/wiki/Gallium.

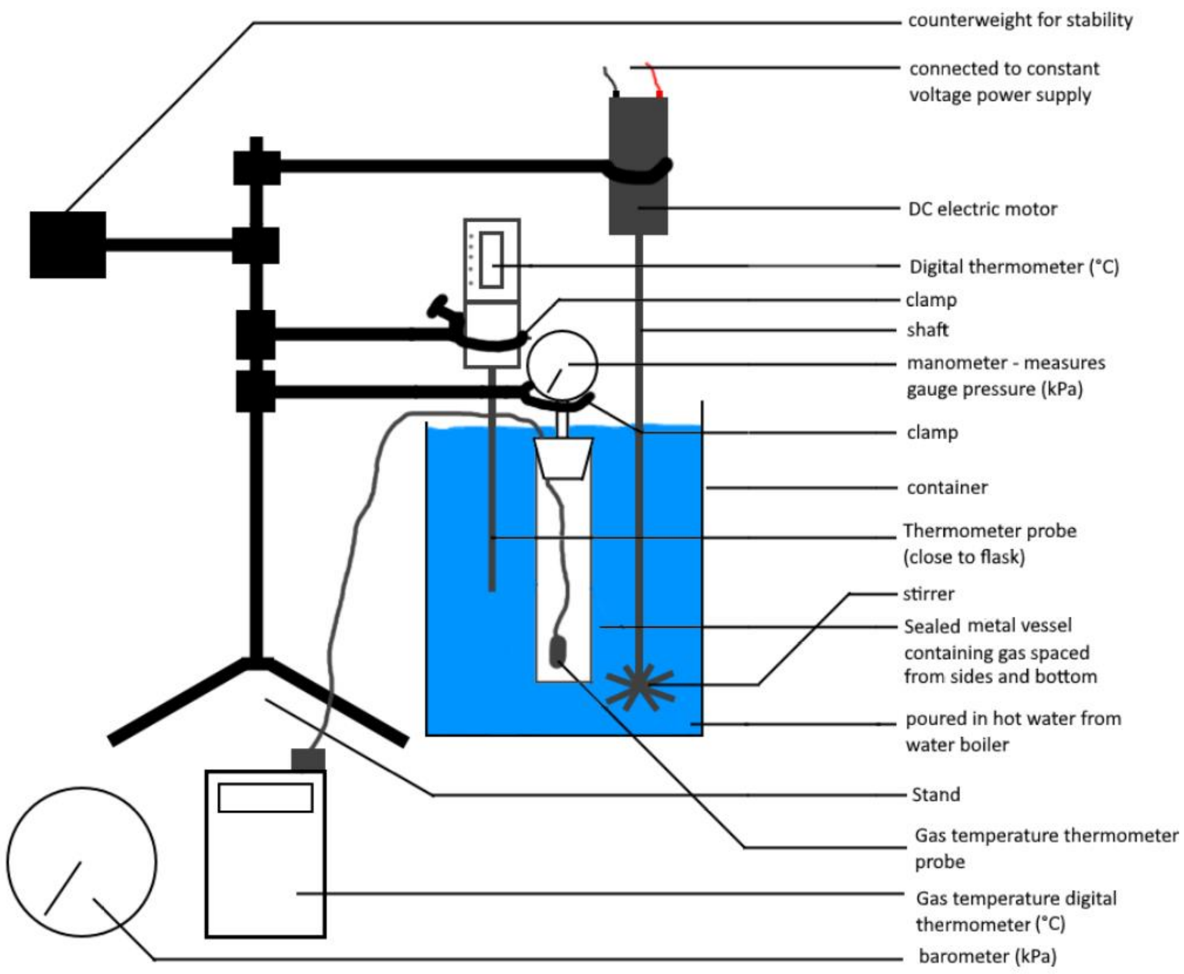

Figure 5 Numerous method improvements

In the calculations aspect, there are also numerous shortfalls. Firstly, the best fit line is determined by manually drawing it. This introduces the possibility for human error, as this process is inherently subjective. A better way to do this would be to determine this line using a mathematical method, such as the least squares regression method[3], eliminating human error in this regard. Both the slope and pressure axis intercept may be calculated using this method. Note that n denotes the number of points:

$$a = \frac{n \cdot \sum(\rho F_b) - \sum(\rho) \cdot \sum(F_b)}{n \cdot \sum(\rho^2) - (\sum(\rho))^2}$$

$$b = \frac{\sum(F_b) - a \sum(\rho)}{n}$$

Although this could be calculated manually, a more practical method would be to use a computer program to do this automatically. In this case, I used Microsoft Excel. Before inputting the data, I added

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[3] "Least Squares Regression." *Mathsisfun.com*, 2017, www.mathsisfun.com/data/least-squares-regression.html.

the atmospheric pressure to all pressure value points as this was previously done later in the procedure to make drawing easier, which is no longer being done. When the data is inputted into Microsoft Excel and the least squares regression method is applied, the following is obtained (Table 3):

	Coefficients	Standard Error
Buoyancy axis intercept	91.8kPa	0.12 kPa
Best fit line gradient	0.334 kPa/°C	0.001 kPa/°C

Table 2 Microsoft Excel regression function output values

The obtained values are very similar to those obtained from manually drawing the graph. Although there are differences, they are within the last significant digit and not very large. Furthermore, the standard errors are very small, meaning that the points do not significantly diverge from the general trend, further affirming a linear relationship. Although the best fit line was drawn correctly in this case, it would be prudent to use this method generally as it eliminates human error.

Another possible issue with the data analysis is the determination of the dependence. Initially, after the graph was drawn, an inverse-proportional or linear/proportional relationship had to be visually determined in order to proceed with the appropriate data analysis. In this case it was linear. However, a very steep section of an inverse-proportional graph may appear linear while it is indeed not. Furthermore, as previously discussed, the temperature range was limiting. A further check may be conducted by drawing a graph in which the pressure axis remains the same and the temperature axis is converted to 1/temperature. Additional computer processing of the data showed that the manually obtained best fit line is very similar to the hand drawn one. If it appears linear or proportional, it indicates that the dependance is inverse proportional, while if it does not appear so it would signify proportion or linearity. In order to simply visually determine this, a graph in Excel was made (used merely for visual determination). It is shown as Figure 6:

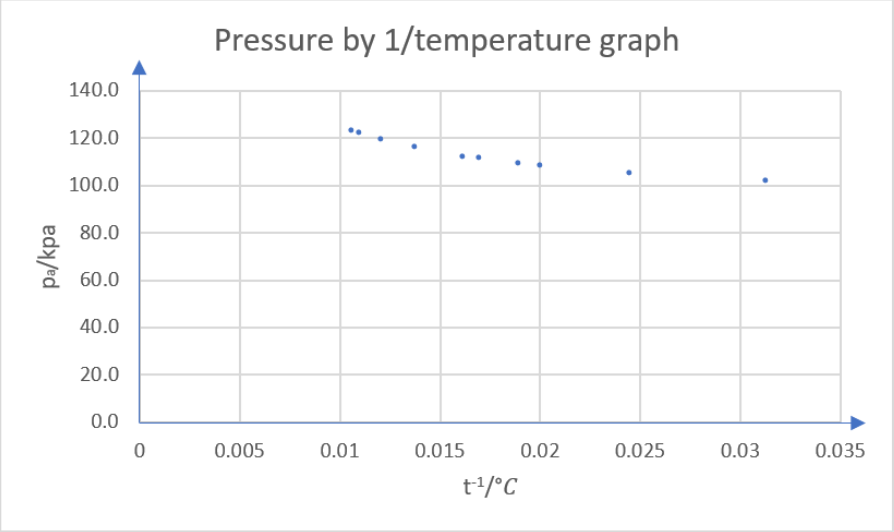

Figure 6 Pressure by 1/temperature graph

The graph does not show linear or proportional dependance, but rather inverse proportional dependence, signifying that the dependance is linear or proportional as initially determined. Same as the previous improvement, methods which eliminate human error should be employed in the future.

There are numerous implications of this dependance in the real world. Although the isochoric process present in this situation is not as common as isothermal and isobaric, it is still present. For example, when a sealed pressure cooker is used and has not yet triggered the pressure release valve, the isochoric process is occurring. In order for the designers to create safe pressure vessels in such situations, the obtained dependance may be employed to determine the maximum temperature at which the vessel may be put before the pressure rises too high for the vessel to handle. This may be applied in numerous industrial processes where a set volume of substance is sealed and heated.

Conclusion

$$p_a = \left(0.341\frac{kPa}{°C}\right) \cdot t + 91.4kPa$$

Through the method, the above equation of dependence of absolute pressure on temperature was obtained. When the equation is converted to apply for degrees Kelvin, a dependence similar to Gay-Lussac's law is obtained, although there are some differences, likely due to issues with the method. Some significant issues with the method were identified which may have influenced the result and improvements were accordingly proposed. The obtained result has many applications which involved sealed vessels with changing temperature, which are common in various industrial processes.

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"DHMZ - Croatian Meteorological and Hydrological Service." *Meteo.hr*, 2021, meteo.hr/naslovnica_aktpod_en.php?tab=aktpod.

Wikipedia Contributors. "Gallium." *Wikipedia*, Wikimedia Foundation, 27 Apr. 2021, en.wikipedia.org/wiki/Gallium.

FIGURE 4. GAUGE PRESSURE - TEMPERATURE (°C) GRAPH